8th International Olympiad on Astronomy and Astrophysics
Suceava - Gura Humorului - August 2014

TEAM COMPETITION

For this test you have to use the materials you found in the box: milimetric paper, plasticine, a ruler, wire, knife, scissors, auto adhesive paper, adhesive band, pins and a support plate.

Defensing the Earth with plasticine, a wire, a pen and scissors

A giant asteroid, potentially dangerous for Earth flight towards the Earth. The collision is inevitable so from an artificial Earth's satellite a nuclear missile will be launched in order to brake into parts the asteroid. You are in a control post, with no computers, only with materials from the box you receive. In one hour and half you have to send the box with the solution of the following problems in order to plan the defensive strategy for our planet.

Real facts
The asteroid 2013 UX11 (Galați – Romania 2013)

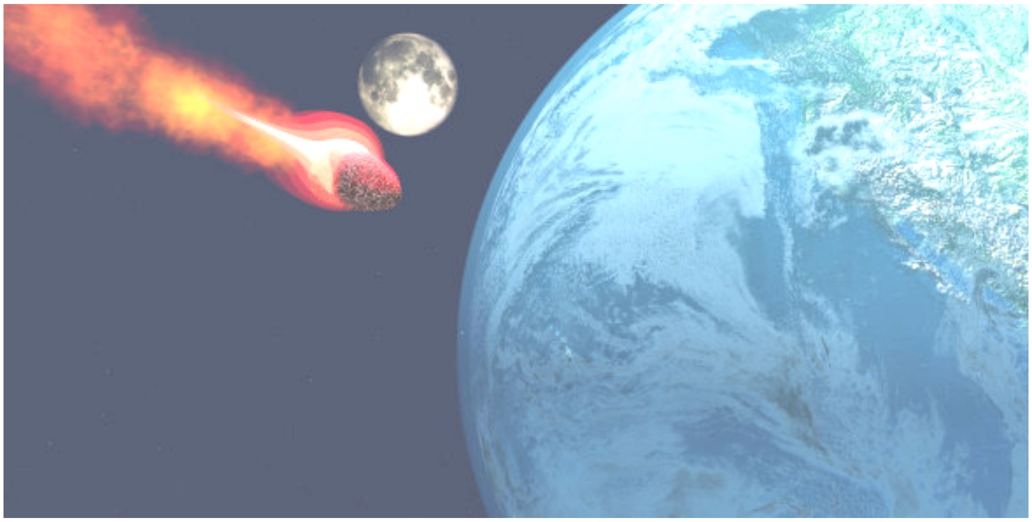
During the night between 29th and 30th of October 2013, two astronomers Ovidiu Tercu and Alex Dumitriu from the Astronomic Observatory from Galati, Romania discovered an asteroid in the Torro constellation. The asteroid named 2013 UX11 has a diameter of $D = 2{,}5\,\text{km}$. This was the first time that an asteroid was discovered by Romanian astronomer in order to make this problem for IOAA. After analyzing the data in the "Minor Planet Center" the following communication was received: ***2013 UX11*** is an asteroid from the Main Belt of Asteroids, orbiting between the orbits of Mars an Jupiter with period $T = 4{,}2\,years$ with an eccentricity $e = 0{,}15$.

a) *Find out* the characteristics of orbit of the ***2013*** UX11 regrdless to the Sun- $(a, b, c, r_{\min}, r_{\max})$. You know the mass of the Sun $M = 9{,}1 \cdot 10^{30}\ \text{kg}$ and the gravitational constant $K = 6{,}67 \cdot 10^{-11}\ \text{Nm}^2\text{kg}^{-2}$.

b) *Find out the maximum and respectively* the minimum of the temperatures of the surface of the asteroid. You know : Temperature of the surface of the Sun $T_S = 6000\,\text{K}$; the radius of the Sun $R_S \approx 7 \cdot 10^5\ \text{km}$; the albedo of the surface of the asteroid $\alpha = 0{,}2$.

The plan

You have received a scaled sketch of the positions of the Earth (P), satellite (S) and the point where the missile have to hit the asteroid. The distance between asteroid and the Earth is $r = 30000\,\text{km}$.



You have to analyze the problem of saving the Earth only by using the materials from the box in the following conditions:

A. For a given speed of the missile v_0, the problem have only one solution i.e. there is only one trajectory form S to A, available for the missile in order to hit the asteroid. You have to determine by using the materials :

a) the elements of the missile trajectory;

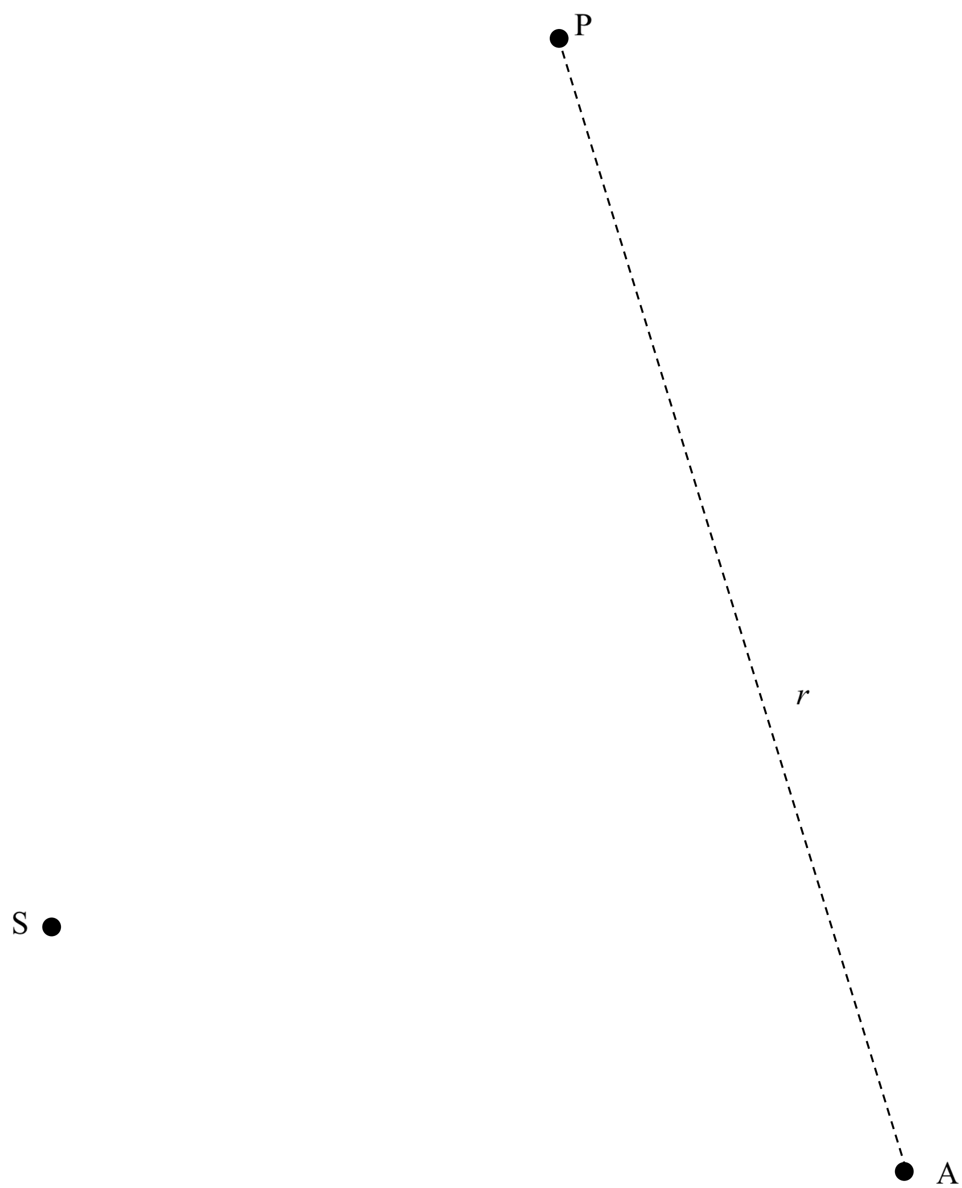
b) the direction of the initial speed of the missile $\vec{v}_0$ in order to be able to hit the asteroid. Indicate on the paper sheet the angle you measured.

c) Calculate the launch velocity of the missile v_0 if you know the mass of the Earth $M = 6 \cdot 10^{24}$ kg and the gravitational constant $K = 6{,}67 \cdot 10^{-11}$ Nm^{2}kg^{-2}

d) Using the plasticine imagine a very simple device in order to calculate the duration of missile flight from launch untill the impact with the asteroid.

e) Using the wire, measure the distance covered by the missile from the launch-point to the impact-point.

A



B. Two solutions . To be sure that the asteroid will be destroyed from another satellite orbiting the Earth at another altitude will be launched in the same moment of time, a second missile in order to hit the asteroid simultaneously with the first missile. Notations P – center of the Earth; S – launch position of the new missile, A – the impact point at distance $r = 30000\ \text{km}$ from the Earth.

In this configuration you know that there are two possible trajectories, form **S** to **A** for the second missile. For that a mysterious point X is marked on the sketch.

You have to *find out*:

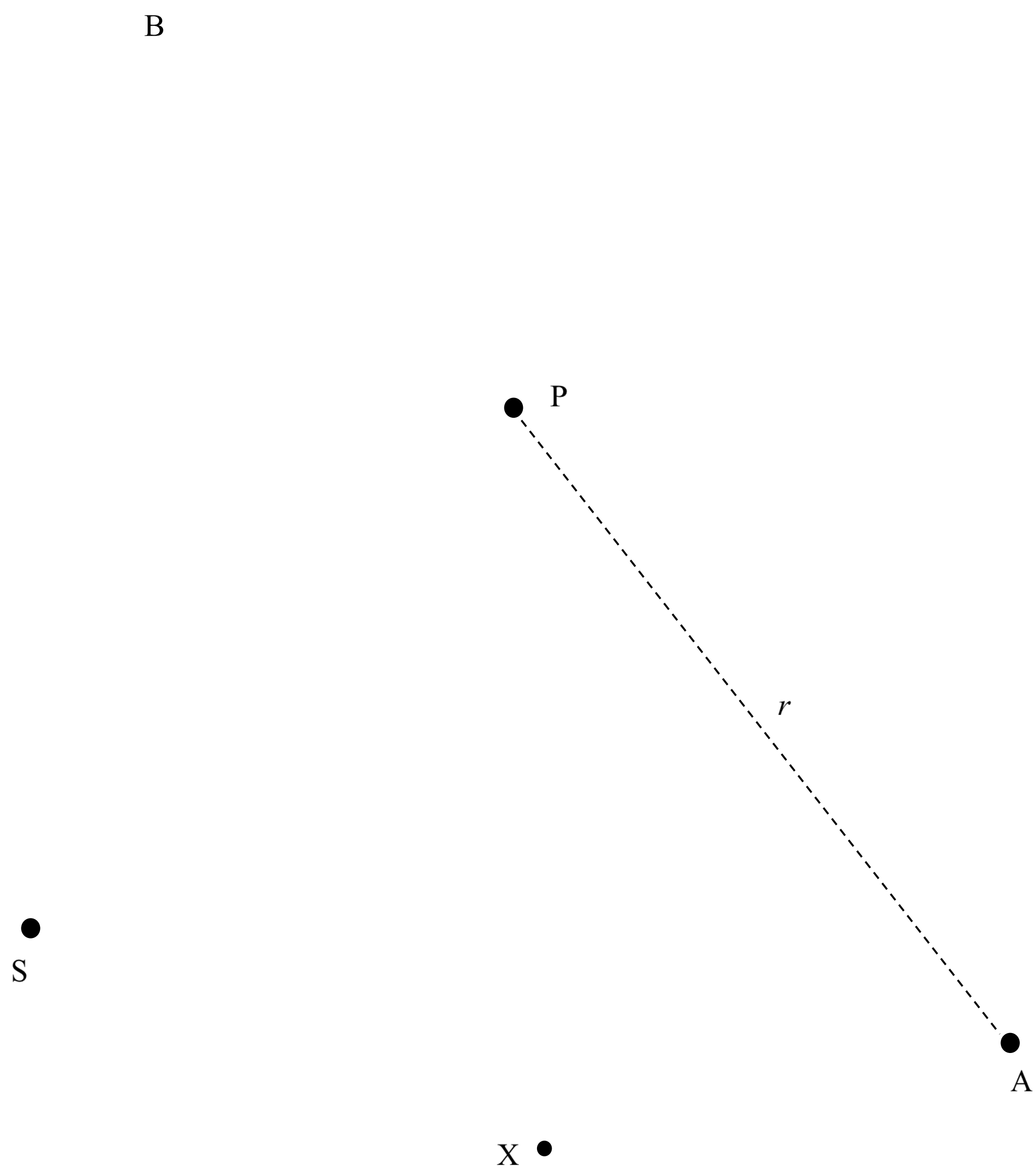
a) the elements of the two possible trajectories.

b) For each trajectory determine the directions of the launch-speeds, at the same velocity which allows the missile to hit the target;

c) The launch- velocity v_0, if you know the maa of the Earth $M = 6 \cdot 10^{24}\ \text{kg}$ and the gravitational constant $G = 6.67 \cdot 10^{-11}\ \text{Nm}^2\text{kg}^{-2}$;

d) Using the plasticine imagine a very simple device in order to calculate the durations of missile flight from launch till the impact with the asteroid for the both trajectories

e) Using the wire, measure the distances covered by the missile from launch-point to the impact-point.


B
P
r
S
A
X

C. Security zone

You have to delimitate around the Earth a security zone. From the point A of this problem you know that there is a unique possible trajectory for the defensive missile. This means that the pointA on the sketch is locate on a curve which delimitates a security yone for the Earth.

a) If you know that the triangle defined by the points **P-** the Earth, **S** – Sun, **A** – impact point remains unchanged in time establish the shape of the curve which delimitates the security zone
b) Draw on the paper this curve and determine its parameters
c) Where have to be situated the impact – point **A** in order to hit the asteroid simultaneously with to missiles
d) Use your plasticine device to fiind out the time spent by an fragment of the asteroid which flies after the impact with the missile, on the curve which delimitates the security zone

Solving

a)

$$T^2 = \frac{4\pi^2}{KM} \cdot a^3;$$

$$a = \sqrt[3]{\frac{T^2 KM}{4\pi^2}};$$

$$T = 4{,}2\,\text{ani} = 4{,}2 \cdot 365 \cdot 24 \cdot 3600\,\text{s};$$

$$a = \sqrt[3]{\frac{(4{,}2 \cdot 365 \cdot 24 \cdot 36)^2 \cdot 10^4 \cdot 6{,}67 \cdot 10^{-11} \cdot 9{,}1 \cdot 10^{30}}{4 \cdot (3{,}14)^2}}\ \text{m};$$

$$a = \sqrt[3]{\frac{(4{,}2 \cdot 365 \cdot 24 \cdot 36)^3 \cdot 10^4 \cdot 6{,}67 \cdot 10^{-11} \cdot 9{,}1 \cdot 10^{30}}{4{,}2 \cdot 365 \cdot 24 \cdot 36 \cdot 4 \cdot (3{,}14)^2}}\ \text{m};$$

$$a = 4{,}2 \cdot 365 \cdot 24 \cdot 36 \cdot \sqrt[3]{\frac{10^4 \cdot 6{,}67 \cdot 10^{-11} \cdot 9{,}1 \cdot 10^{30}}{4{,}2 \cdot 365 \cdot 24 \cdot 36 \cdot 4 \cdot (3{,}14)^2}}\ \text{m};\ (3{,}14)^2 \approx 10;$$

$$a \approx 4{,}2 \cdot 365 \cdot 24 \cdot 36 \cdot \sqrt[3]{\frac{6{,}67 \cdot 9{,}1 \cdot 10^{22}}{4{,}2 \cdot 365 \cdot 24 \cdot 36 \cdot 4}}\ \text{m};$$

$$a \approx 1324512 \cdot \sqrt[3]{\frac{60697000 \cdot 10^{15}}{5298048}}\ \text{m};$$

$$a \approx 1324512 \cdot \sqrt[3]{11{,}4564836} \cdot 10^5\ \text{m};\ \sqrt[3]{11{,}4564836} \approx 2{,}25;$$

$$a \approx 1324512 \cdot 2{,}25 \cdot 10^5\ \text{m} = 2980152 \cdot 10^5\ \text{m};$$

$$a \approx 3 \cdot 10^{11}\ \text{m} = 3 \cdot 10^8\ \text{km};$$

$$e = \sqrt{1 - \frac{b^2}{a^2}};\ b = a \cdot \sqrt{1 - e^2};$$

$$b = 3 \cdot 10^8 \cdot \sqrt{1 - (0{,}15)^2}\ \text{km};\ b \approx 2{,}96 \cdot 10^8\ \text{km};$$

$$c = \sqrt{a^2 - b^2} \approx 0{,}488 \cdot 10^8\ \text{km};$$

$$r_{\min} = a - c = 2{,}512 \cdot 10^8\ \text{km};$$

$$r_{\max} = a + c = 3{,}488 \cdot 10^8\ \text{km}.$$

b.

$$Q_{\text{Soare}} = \frac{E_{\text{emis,Soare}}}{t S_{\text{Soare}}} = \sigma T_S^4,$$

unde σ este o constantă;

$$\frac{E_{\text{emis,Soare}}}{t} = P_{\text{emis,Soare}};\ \sigma T_{\text{S}}^{4} = \frac{P_{\text{emis,Soare}}}{4\pi R_{\text{S}}^{2}};$$

$$P_{\text{emis,Soare}} = \sigma T_{\text{S}}^{4} 4\pi R_{\text{S}}^{2}.$$

Densitatea fluxului energetic al Soarelui, la distanţa r_{AS} faţă de acesta (acolo unde se află Asteroidul), însemnează energia tuturor radiaţiilor emise de Soare, care traversează unitatea de arie a unei suprafeţe, sub incidenţă normală, în unitatea de timp, adică:

$$\phi_{\text{Soare},r_{\text{AS}}} = \frac{E_{\text{emis,Soare}}}{St} = \frac{\frac{E_{\text{emis,Soare}}}{t}}{S} = \frac{P_{\text{emis,Soare}}}{S} = \frac{P_{\text{emis,Soare}}}{4\pi r_{\text{AS}}^{2}};$$

$$\phi_{\text{Soare},r_{\text{AS}}} = \frac{\sigma T_{\text{S}}^{4} 4\pi R_{\text{S}}^{2}}{4\pi r_{\text{AS}}^{2}}.$$

Semisfera asteroidului expusă radiaţiilor solare este echivalentă cu un disc plan circular, având raza R_{A} şi aria suprafeţei πR_{A}^{2}, aşezat perpendicular pe direcţia Soare – Asteroid, astfel încât fluxul radiaţiilor solare incidente, F, la nivelul Asteroidului (adică energia solară incidentă le nivelul Asteroidului, în unitatea de timp), este:

$$F_{\text{incident}} = \phi_{\text{Soare},r_{\text{AS}}} \cdot \pi R_{\text{A}}^{2} = P_{\text{incident}} = P_{\text{emis,Soare}};$$

$$F_{\text{incident}} = \frac{\sigma T_{\text{S}}^{4} 4\pi R_{\text{S}}^{2}}{4\pi r_{\text{AS}}^{2}} \cdot \pi R_{\text{A}}^{2} = P_{\text{incident}};$$

$$\alpha = \frac{P_{\text{reflectat,Asteroid}}}{P_{\text{incident}}};$$

$$P_{\text{reflectat,Asteroid}} = \alpha P_{\text{incident}} = \alpha \frac{\sigma T_{\text{S}}^{4} 4\pi R_{\text{S}}^{2}}{4\pi r_{\text{AS}}^{2}} \cdot \pi R_{\text{A}}^{2}.$$

În acord cu desenul din figura alăturată, ecuația bilanțului energetic al procesului analizat este:

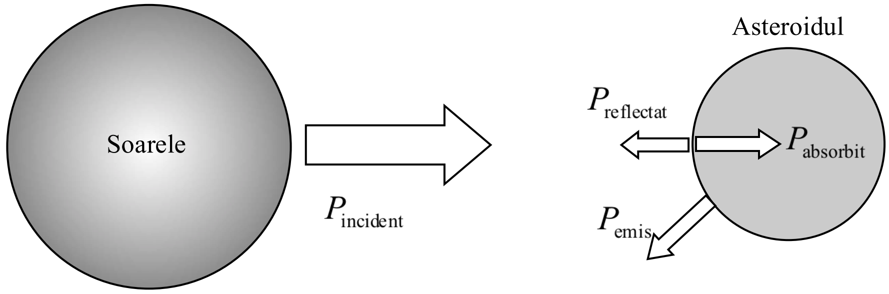


Fig.

$$P_{\text{incident,Asteroid}} = P_{\text{reflectat,Asteroid}} + P_{\text{absorbitAsteroid}};$$

$$\frac{\sigma T_{\text{S}}^{4} 4\pi R_{\text{S}}^{2}}{4\pi r_{\text{AS}}^{2}} \cdot \pi R_{\text{A}}^{2} = \alpha \frac{\sigma T_{\text{S}}^{4} 4\pi R_{\text{S}}^{2}}{4\pi r_{\text{AS}}^{2}} \cdot \pi R_{\text{A}}^{2} + P_{\text{absorbit,Asteroid}}.$$

$$P_{\text{emis,Asteroid}} = \sigma T_A^4 \cdot 4\pi R_A^2.$$

Stationary

$$P_{\text{emis,Asteroid}} = P_{\text{absorbit,Asteroid}} = \sigma T_A^4 \cdot 4\pi R_A^2,$$

$$\frac{\sigma T_S^4 4\pi R_S^2}{4\pi r_{AS}^2} \cdot \pi R_A^2 = \alpha \frac{\sigma T_S^4 4\pi R_S^2}{4\pi r_{AS}^2} \cdot \pi R_A^2 + \sigma T_A^4 \cdot 4\pi R_A^2;$$

$$\frac{T_S^4 R_S^2}{4 r_{AS}^2} = \alpha \frac{T_S^4 R_S^2}{4 r_{AS}^2} + T_A^4;$$

$$(1-\alpha)\frac{T_S^4 R_S^2}{4 r_{AS}^2} = T_A^4;$$

$$T_A = T_S \cdot \sqrt[4]{1-\alpha} \cdot \sqrt{\frac{R_S}{2 r_{AS}}},$$

Rezults

$$T_{A,\max} = T_S \cdot \sqrt[4]{1-\alpha} \cdot \sqrt{\frac{R_S}{2 r_{AS,\min}}};$$

$$T_S = 6000\ \text{K};\ R_S \approx 7 \cdot 10^5\ \text{km};\ r_{AS,\min} = 2{,}512 \cdot 10^8\ \text{km};\ \alpha \approx 0{,}2;$$

$$T_{A,\max} = 6000\ \text{K} \cdot \sqrt[4]{1-0{,}2} \cdot \sqrt{\frac{7 \cdot 10^5\ \text{km}}{2 \cdot 2{,}512 \cdot 10^8\ \text{km}}};$$

$$\sqrt[4]{1-0{,}2} = \sqrt[4]{0{,}8} \approx 0{,}945;$$

$$6000 \cdot \sqrt{\frac{7 \cdot 10^5}{2 \cdot 2{,}512 \cdot 10^8}} = \sqrt{\frac{36 \cdot 7 \cdot 10^3}{2 \cdot 2{,}512}} \approx 223{,}96;$$

$$T_{A,\max} = 223{,}96 \cdot 0{,}945\ \text{K} \approx 211{,}6422\ \text{K},$$

When the astheroid is at perihelium ;

$$T_{A,\min} = T_S \cdot \sqrt[4]{1-\alpha} \cdot \sqrt{\frac{R_S}{2 r_{AS,\max}}};$$

$$T_S = 6000\ \text{K};\ R_S \approx 7 \cdot 10^5\ \text{km};\ r_{AS,\max} = 3{,}488 \cdot 10^8\ \text{km};\ \alpha \approx 0{,}2;$$

$$T_{A,\min} = 6000\ \text{K} \cdot \sqrt[4]{1-0{,}2} \cdot \sqrt{\frac{7 \cdot 10^5\ \text{km}}{2 \cdot 3{,}488 \cdot 10^8\ \text{km}}};$$

$$\sqrt[4]{1-0{,}2} = \sqrt[4]{0{,}8} \approx 0{,}945;$$

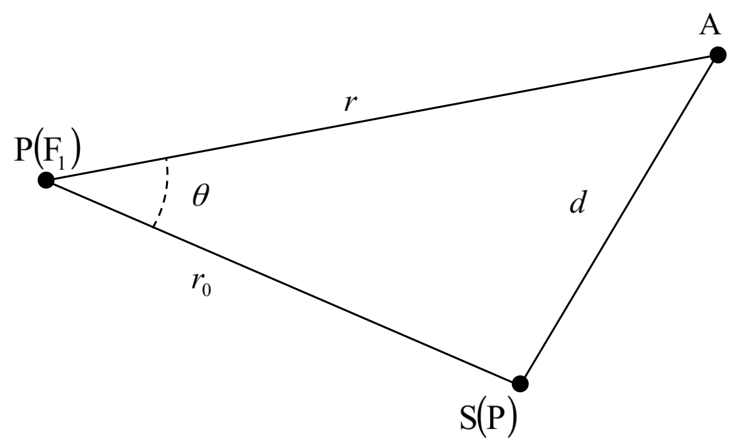
$$6000 \cdot \sqrt{\frac{7 \cdot 10^5}{2 \cdot 3{,}488 \cdot 10^8}} = \sqrt{\frac{36 \cdot 7 \cdot 10^3}{2 \cdot 3{,}488}} \approx 190{,}06;$$

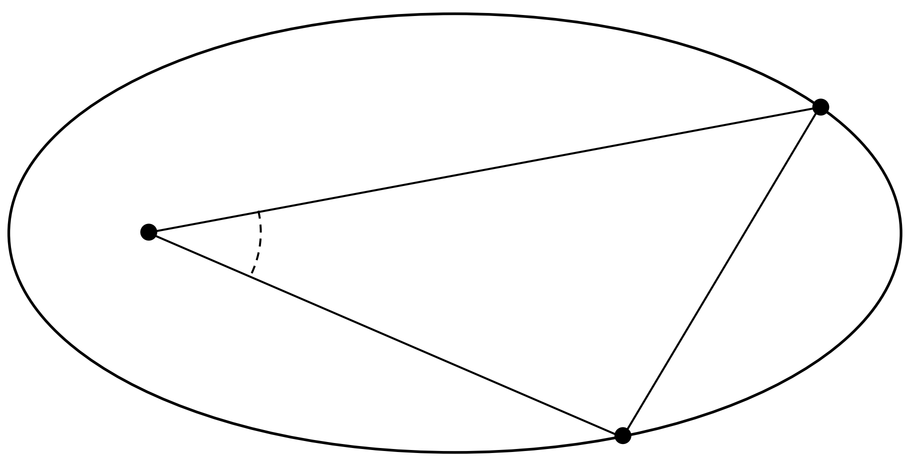
$$T_{A,\min} = 190{,}06 \cdot 0{,}945\ \text{K} \approx 179{,}60\ \text{K},$$

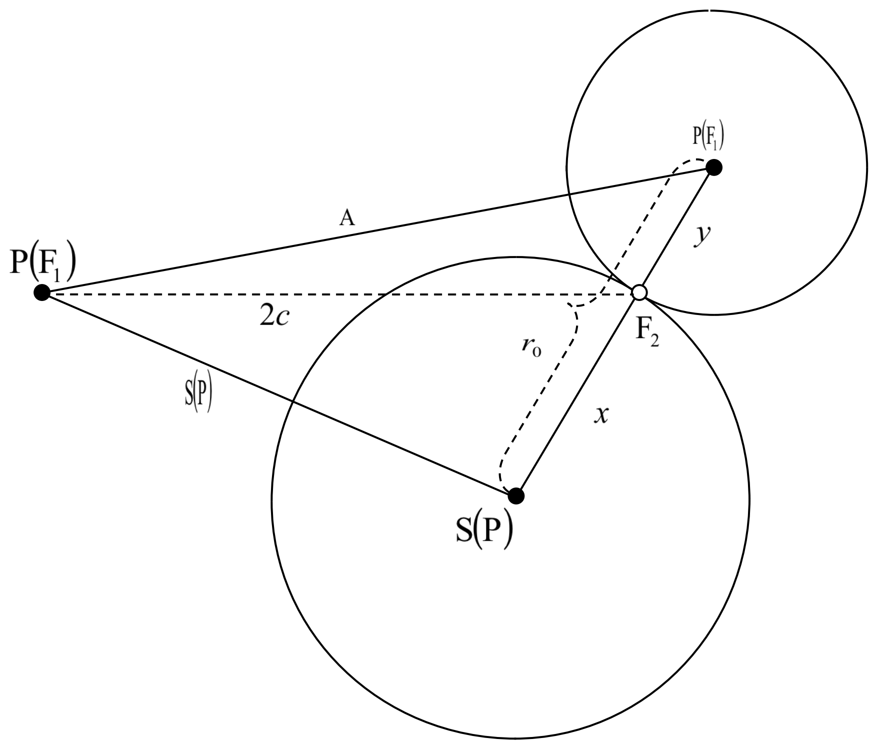
When the astheroid is at aphelium.

A.

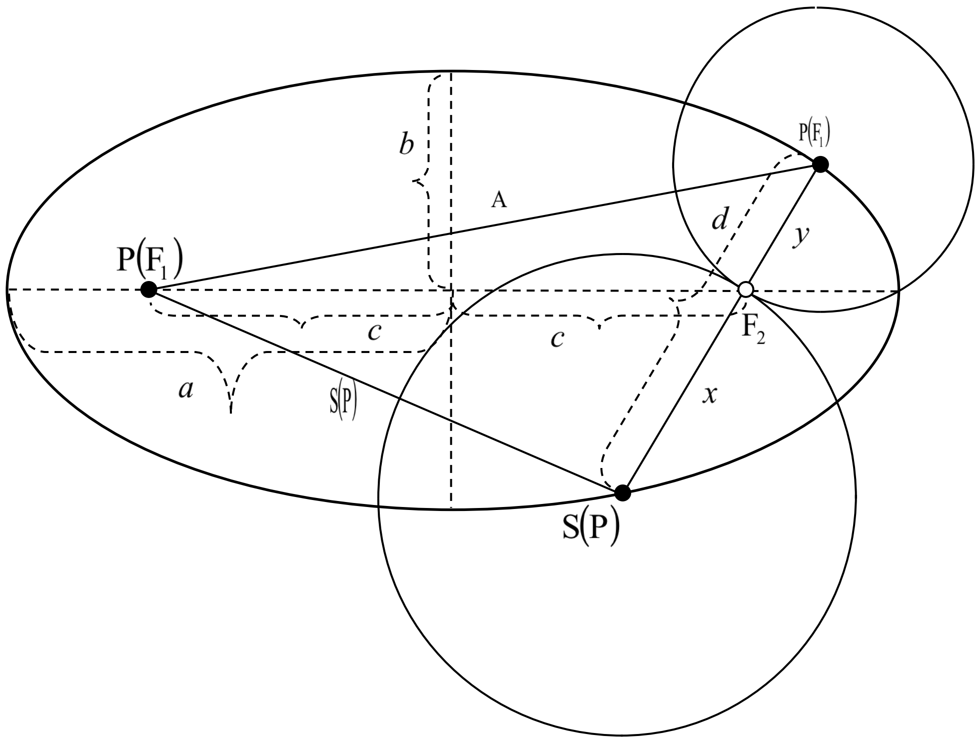
a)







$$r_0 + x = 2a;\ r + y = 2a;$$
$$r_0 + x = r + y;$$
$$x - y = r - r_0 = \Delta r;$$
$$x + y = d;$$
$$x = \frac{1}{2}(d + \Delta r);\ y = \frac{1}{2}(d - \Delta r);$$
$$a = \frac{1}{2}(r_0 + x) = \frac{1}{2}\left[r_0 + \frac{1}{2}(d + \Delta r)\right] = \frac{1}{2}\left[r_0 + \frac{1}{2}(d + r - r_0)\right];$$
$$a = \frac{1}{2}\left[r_0 + \frac{1}{2}(d + r) - \frac{r_0}{2}\right] = \frac{1}{4}(r_0 + r + d);$$
$$2a = \frac{1}{2}(r_0 + r + d);$$
$$F_1F_2 = 2c;$$
$$c = \sqrt{a^2 - b^2};\ b = \sqrt{a^2 - c^2};$$
$$e = \sqrt{1 - \frac{b^2}{a^2}};\ e = \frac{r_{max} - r_{min}}{r_{max} + r_{min}};$$
$$r_{min} = a(1 - e);\ r_{max} = a(1 + e);\ r_{min} + r_{max} = 2a.$$



Din măsurători efectuate pe desen şi din calcule, rezultă:

$$r_0 = 133\text{ mm}; r = 153\text{ mm}; \; d = 115\text{ mm};$$

$$\Delta r = r - r_0 = 20\text{ mm};$$

$$x = 67{,}5\text{ mm}; \; y = 47{,}5\text{ mm};$$

$$2a = 200\text{ mm}; \; a = 100\text{ mm};$$

$$2b = 148\text{ mm}; \; b = 74\text{ mm};$$

$$2c = 134\text{ mm}; \; c = 67\text{ mm};$$

$$e \approx 0{,}67; \; r_{\min} = 33\text{ mm}; \; r_{\max} = 167\text{ mm};$$

$$r_{\text{real}} = 30000\text{ km}; \; r = 153\text{ mm};$$

$$S = \frac{r_{\text{real}}}{r} = \frac{30000}{153}\frac{\text{km}}{\text{mm}};$$

$$r_{0,\text{real}} = r_0 S \approx 26078{,}43\text{ km}; \; d_{\text{real}} = dS \approx 22549\text{ km};$$

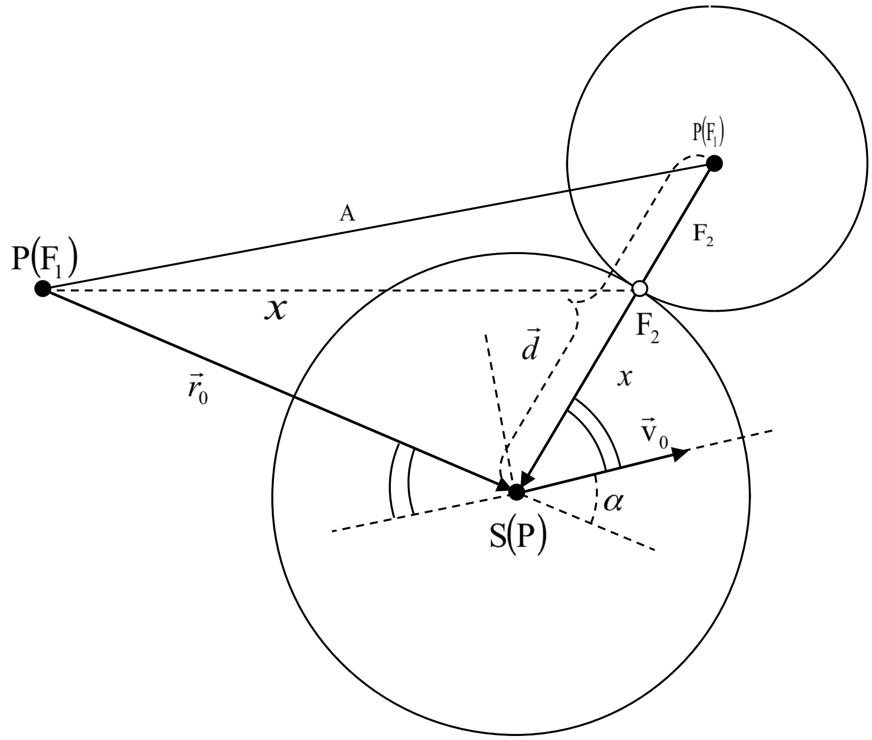
$$x_{\text{real}} = xS \approx 13235{,}3\text{ km}; \; y_{\text{real}} = yS \approx 9313{,}7\text{ km};$$

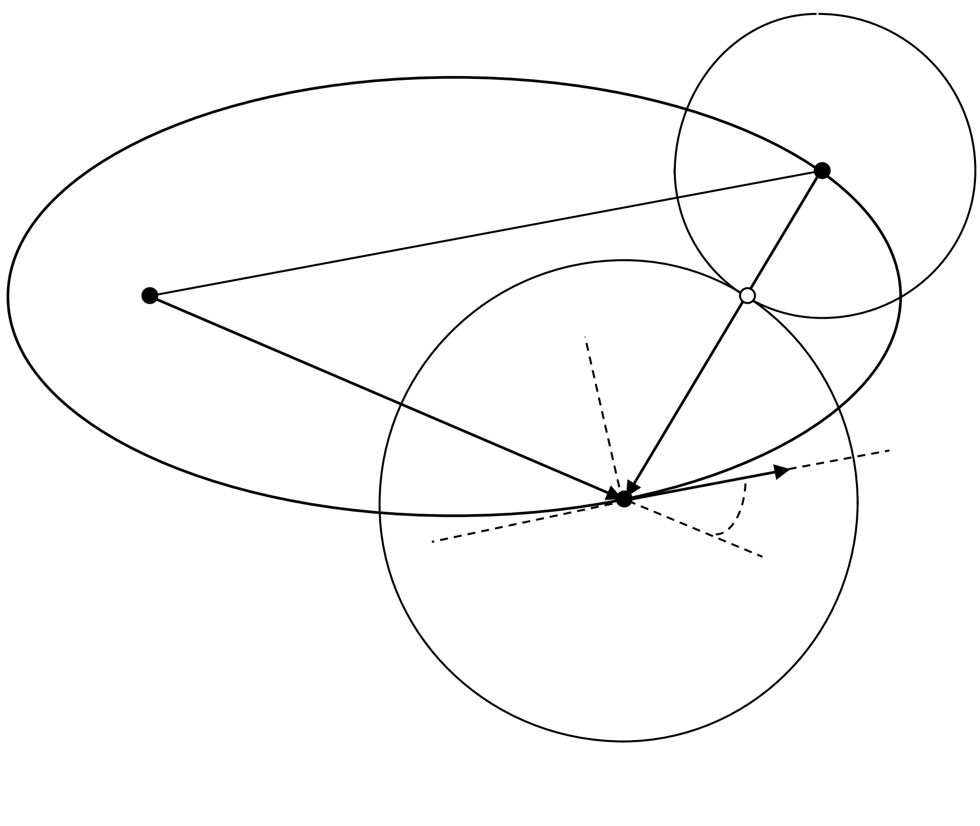
$$a_{\text{real}} \approx 19607{,}84\text{ km}; \; b_{\text{real}} \approx 14509{,}80\text{ km}; \; c_{\text{real}} \approx 13137{,}25\text{ km};$$

$$r_{\min,\text{real}} \approx 6470{,}58\text{ km}; \; r_{\max,\text{real}} \approx 32745{,}09\text{ km}.$$

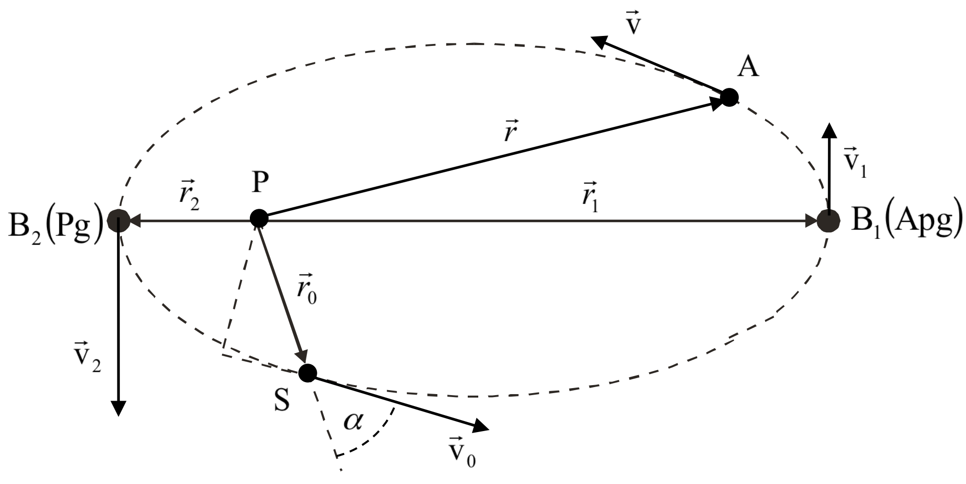
b)
:

$$\alpha \approx 54^0.$$





c)



:

$$\frac{v_{1,2}^2}{2} - K\frac{M}{r_{1,2}} = \frac{v_0^2}{2} - K\frac{M}{r_0};$$

$$v_{1,2}\, r_{1,2} = v_0\, r_0 \sin\alpha;$$

$$\left(\frac{v_0^2}{2} - K\frac{M}{r_0}\right) r_{1,2}^2 + KMr_{1,2} - \frac{1}{2} v_0^2 r_0^2 \sin^2\alpha = 0.$$

r_1 Apogee
r_2, Perigee,:

$$a = \frac{1}{2}(r_1 + r_2),$$

$$a = \frac{KMr_0}{2KM - r_0 v_0^2},$$

relație independent of α relationship for $\vec{v}_0$

$$v_0 = \sqrt{KM\frac{2a_{\text{real}} - r_{0,\text{real}}}{a_{\text{real}} r_{0,\text{real}}}} = \sqrt{\frac{KM}{r_{0,\text{real}}} \cdot \frac{2a_{\text{real}} - r_{0,\text{real}}}{a_{\text{real}}}},$$

$$S = \frac{30000\,\text{km}}{153\,\text{mm}};$$

$$r_{0,\text{real}} = 133\,\text{mm} \cdot \frac{30000\,\text{km}}{153\,\text{mm}} \approx 26078\,\text{km};$$

$$M = 6 \cdot 10^{24}\,\text{kg};\; K = 6{,}67 \cdot 10^{-11}\,\text{Nm}^2\text{kg}^{-2};$$

$$2a_{\text{real}} = 200\,\text{mm}\cdot S;$$

$$v_0 = \sqrt{\frac{6{,}67\cdot 10^{-11}\ \text{Nm}^2\text{kg}^{-2}\cdot 6\cdot 10^{24}\ \text{kg}}{26078\cdot 10^3\ \text{m}}\cdot\frac{200-133}{100}};$$

$$v_0 = \sqrt{\frac{6{,}67\cdot 10^{-11}\cdot 6\cdot 10^{24}}{26078\cdot 10^3}\cdot\frac{200-133}{100}}\ \frac{\text{m}}{\text{s}} \approx 3200\,\frac{\text{m}}{\text{s}} = 3{,}2\,\frac{\text{km}}{\text{s}}.$$

d)

$$T^2 = \frac{4\pi^2}{K(M+m)}\cdot a_{\text{real}}^3;\ m << M;\ T^2 = \frac{4\pi^2}{KM}\cdot a_{\text{real}}^3;$$

$$M = 6\cdot 10^{24}\ \text{kg};\ \ K = 6{,}67\cdot 10^{-11}\ \text{Nm}^2\text{kg}^{-2};$$

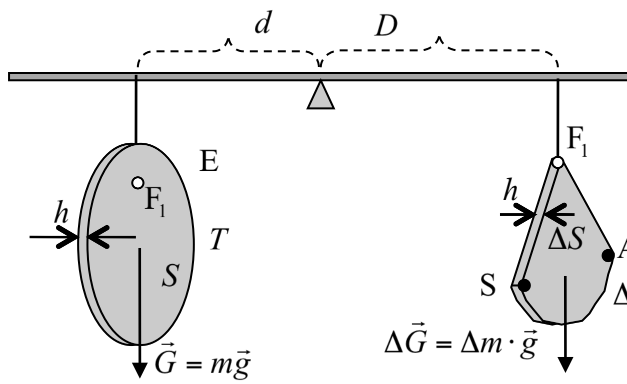
$$S = \frac{30000\,\text{km}}{153\,\text{mm}};$$

$$a_{\text{real}} = 100\,\text{mm}\cdot S \approx 19608\ \text{km};$$

$$T = 2\pi\sqrt{\frac{a_{\text{real}}^3}{KM}} = 2\cdot 3{,}14\sqrt{\frac{19608^3\cdot 10^9}{6{,}67\cdot 10^{-11}\cdot 6\cdot 10^{24}}}\ \text{s} \approx 27240\,\text{s};$$

$$T \approx 454\,\text{min} \approx 7{,}56\ \text{h}.$$

The system has to be done



$$G\cdot d = \Delta G\cdot D;\ mg\cdot d = \Delta m\cdot g\cdot D;$$

$$m\cdot d = \Delta m\cdot D;\ \rho\cdot V\cdot d = \rho\cdot\Delta V\cdot D;$$

$$V\cdot d = \Delta V\cdot D;\ S\cdot h\cdot d = \Delta S\cdot h\cdot D;$$

$$S\cdot d = \Delta S\cdot D;\ \frac{\Delta S}{S} = \frac{d}{D}.$$

According to the second Keppler law:

$$T\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots S;$$

$$\Delta t \ldots\ldots\ldots\ldots\ldots\ldots\ldots \Delta S;$$

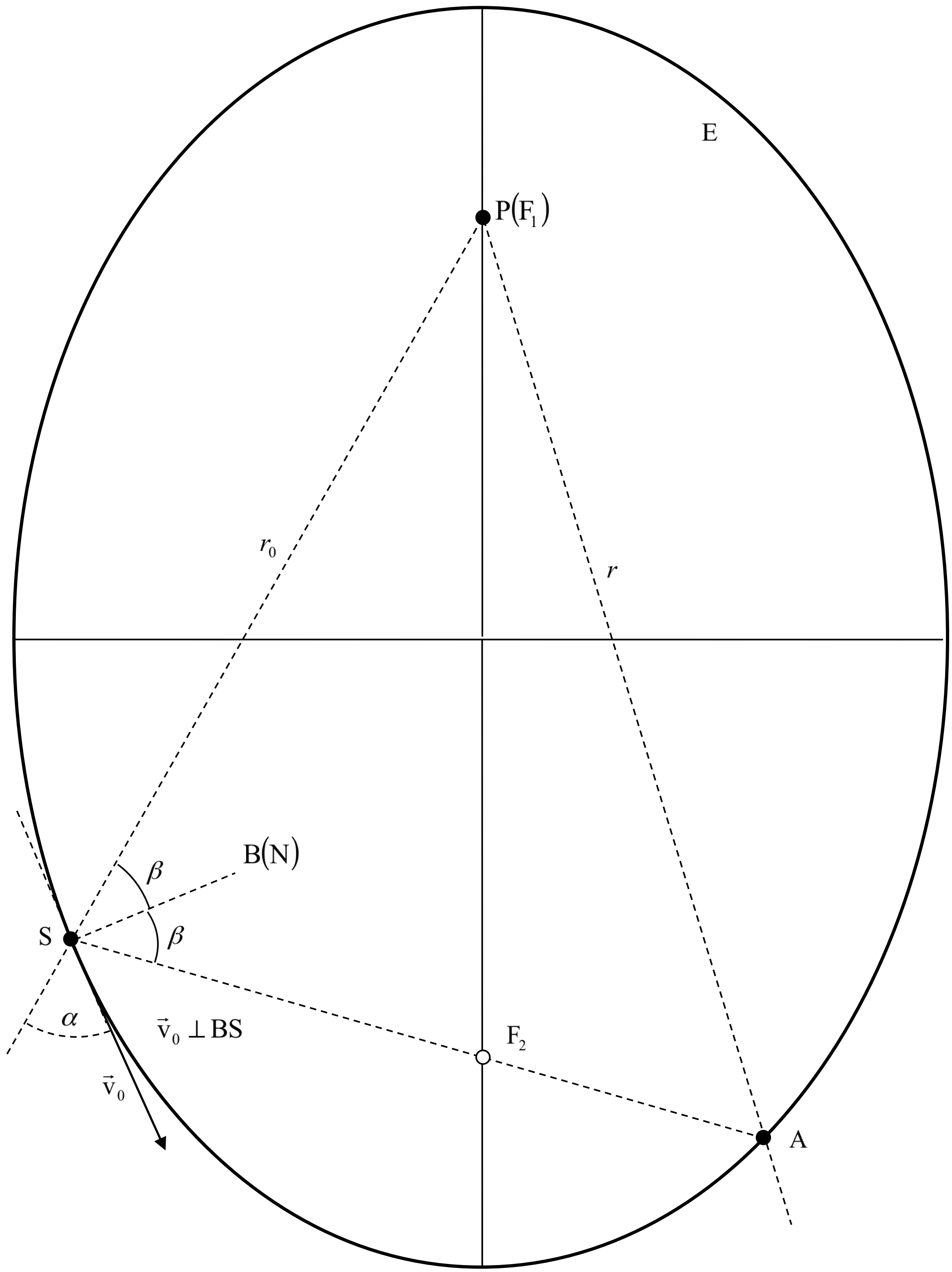
$$\Delta t = \frac{\Delta S}{S} \cdot T = \frac{d}{D} \cdot T,$$

D	d	T	Δt
26 cm	11 cm	7,56 h	3,19 h

e) Measurements ao a wire along the elipse sector between S and A:

$$l_{\mathrm{SA}} = 135\,\mathrm{mm};$$

$$l_{\mathrm{SA,real}} = l_{\mathrm{SA}} \cdot S = 135\,\mathrm{mm} \cdot \frac{30000\,\mathrm{km}}{153\,\mathrm{mm}} \approx 26470{,}58\,\mathrm{km}.$$

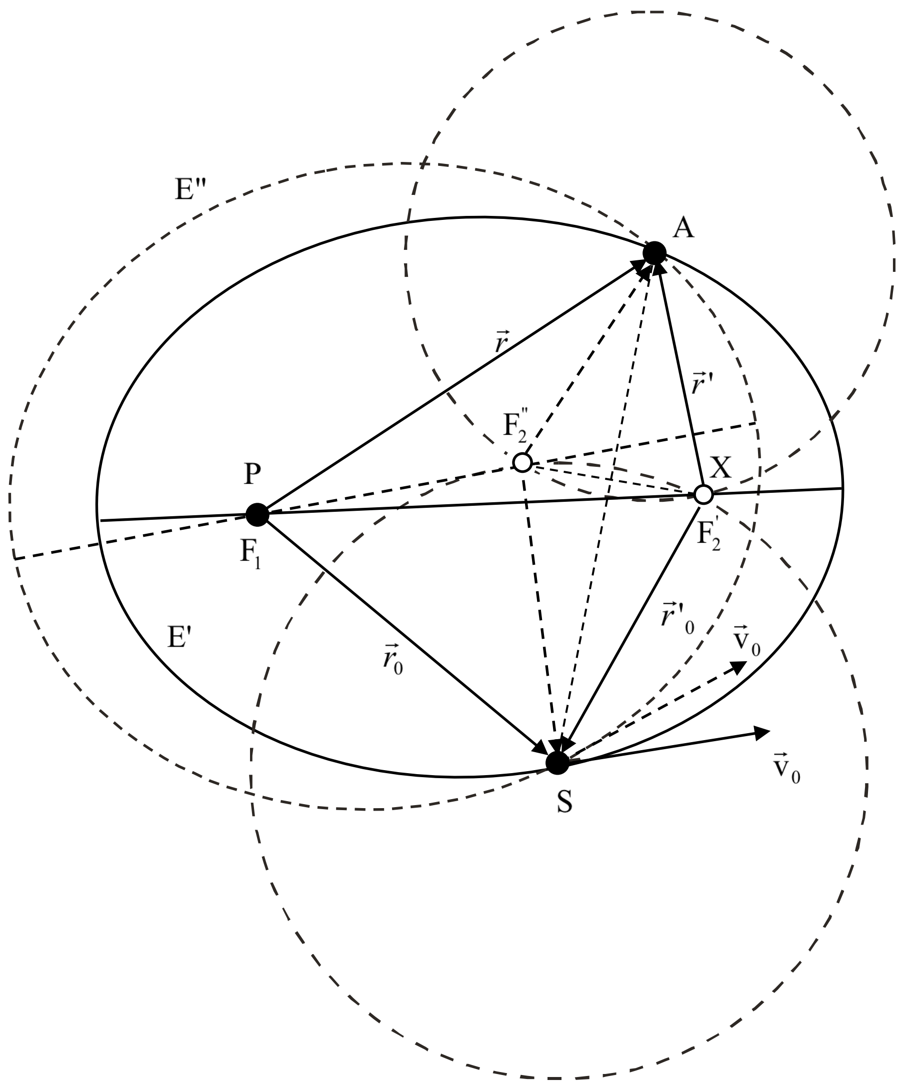

E
P(F1)
r0
r
B(N)
β
β
S
α
v0 ⊥ BS
v0
F2
A

B.

a) The 2 elipses have a common focar so the injection point will go in the same point $\vec{r}_0$, with initial velocity, v_0. **The both elipses semiaxes are identical**:

$$a = \frac{KMr_0}{2KM - r_0\mathrm{v}_0^2}.$$

$2c_1 \neq 2c_2;, \ 2b_1 \neq 2b_2; \ e_1 \neq e_2.$



$$\mathrm{PS} + \mathrm{SX} = \mathrm{PA} + \mathrm{AX} = 170\,\mathrm{mm},$$

$\mathrm{X} \equiv \mathrm{F}_2'$.

AAfter localizing the focuses of the elipses the axes can be drawn.

$$r_0 + r_0' = 2a; r + r' = 2a.$$

Measuring r_0 and r_0', or r and r', can be calculated:

$$S = \frac{30000\ \text{km}}{107\ \text{mm}};$$

$$r_0 = 95\ \text{mm}\cdot S;\ r_0' = 75\ \text{mm}\cdot S;$$

$$r = 107\ \text{mm}\cdot S;\ r' = 63\ \ \text{mm}\cdot S;$$

$$2a = 170\ \text{mm}\cdot S \approx 47664\ \text{km};$$

$$a = \frac{1}{2}\left(r_0 + r_0'\right) = \frac{1}{2}\left(r + r'\right) = 85\ \text{mm}\cdot S = 85\ \text{mm}\cdot\frac{30000\ \text{km}}{107\ \text{mm}} \approx 23832\ \text{km}.$$

$$\text{F}_1\text{F}_2' = 2c_1;\ b_1 = \sqrt{a^2 - c_1^2};$$

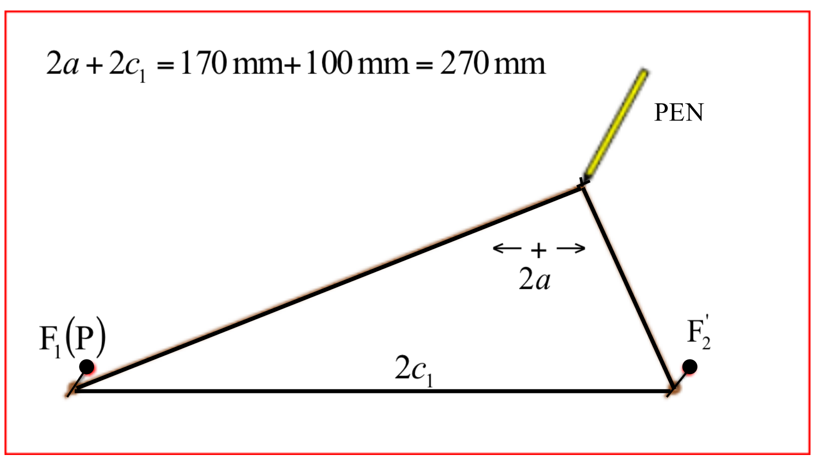
$$2c_1 = 100\ \text{mm}\cdot S;\ c_1 = 50\ \text{mm}\cdot S;$$

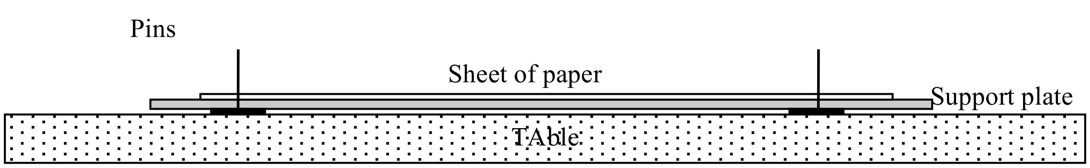
$$b_1 \approx 69\ \text{mm}\cdot S \approx 19346\ \text{km};$$

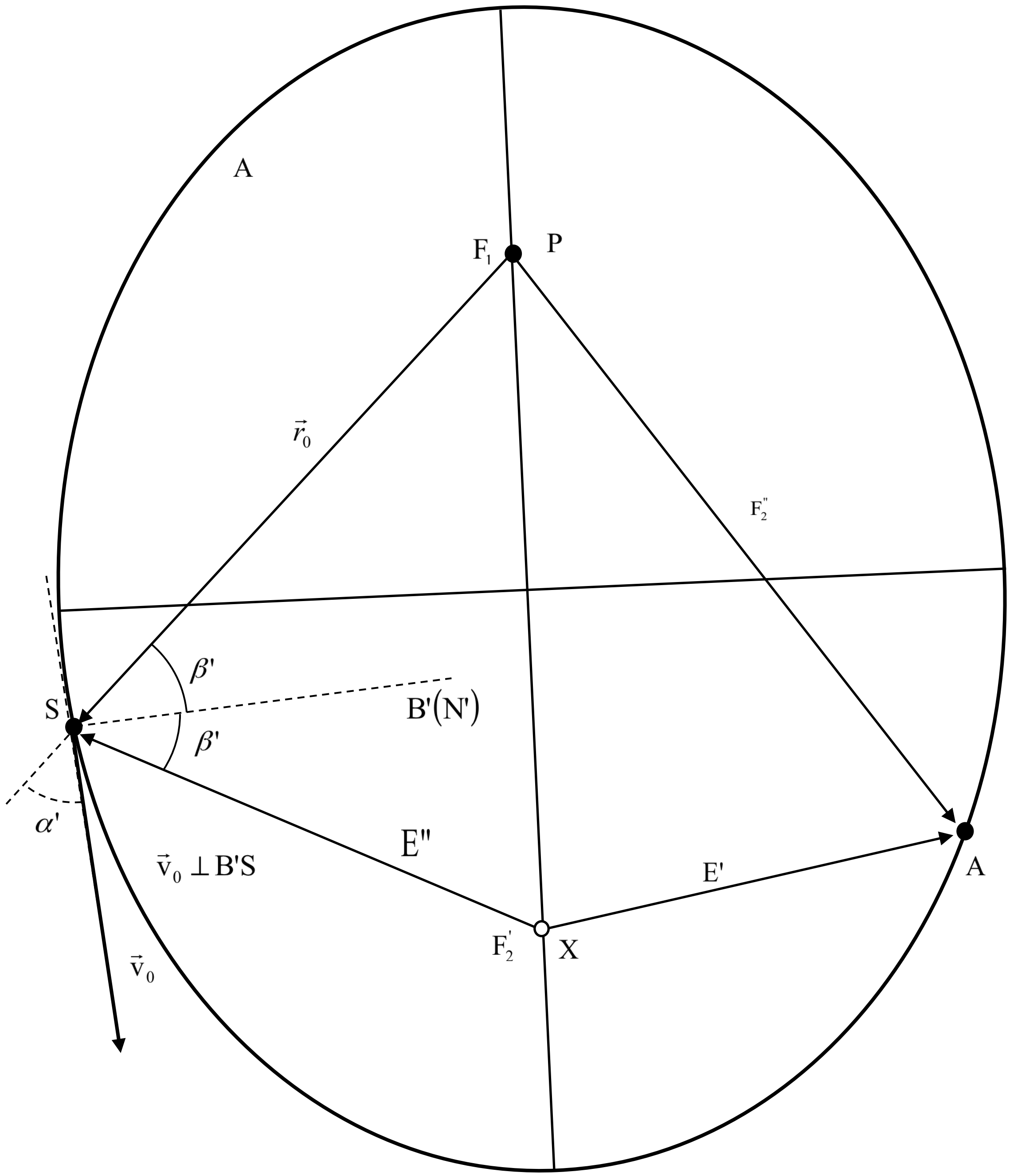
$$\text{F}_1\text{F}_2'' = 2c_2;\ b_2 = \sqrt{a^2 - c_2^2};$$

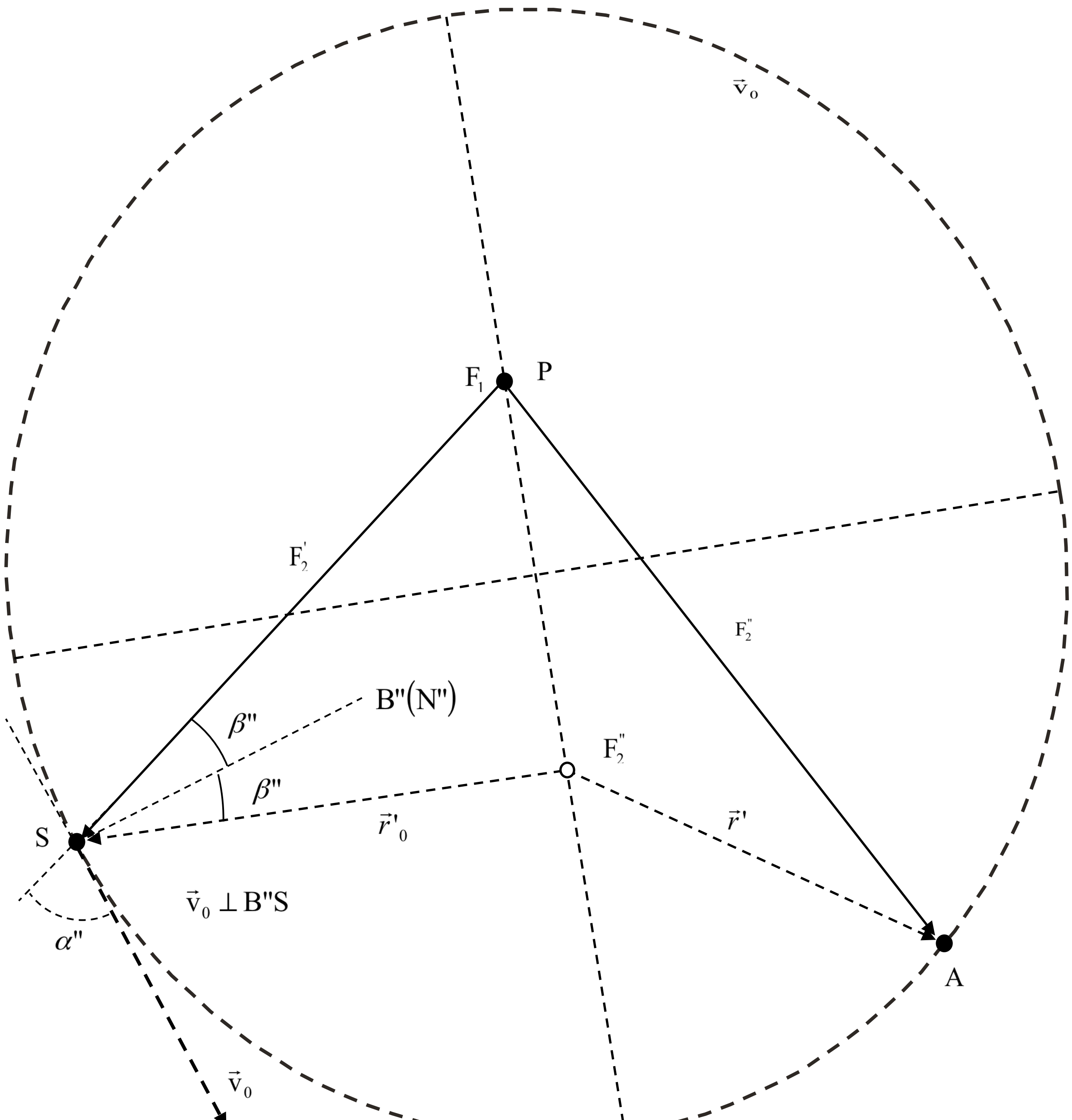
$$2c_2 = 60\ \text{mm}\cdot S;\ c_2 = 30\ \text{mm}\cdot S;$$

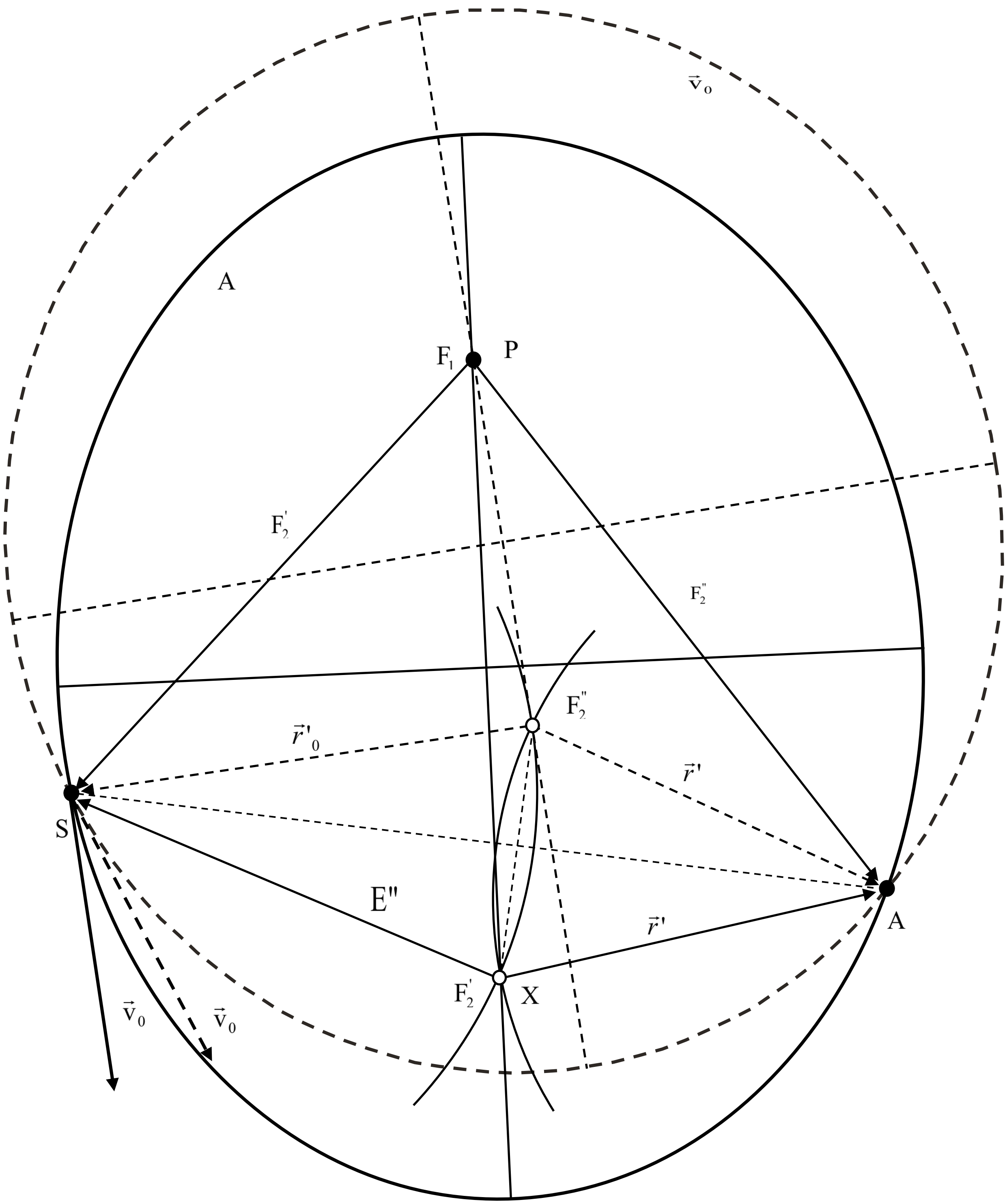
$$b_2 \approx 80\ \text{mm}\cdot S \approx 22430\ \text{km}.$$






A
F1
P
$\vec{r}_0$
F_2''
S
β'
B'(N')
β'
α'
E''
$\vec{v}_0 \perp B'S$
E'
A
F_2'
X
$\vec{v}_0$


$\vec{v}_0$
F_1
P
F_2'
F_2''
$B''(N'')$
β''
β''
F_2''
S
$\vec{r}'_0$
$\vec{r}'$
$\vec{v}_0 \perp B''S$
α''
A
$\vec{v}_0$


$\vec{v}_o$
A
F_1
P
F_2'
F_2''
F_2''
$\vec{r}'_0$
$\vec{r}'$
S
E''
$\vec{r}'$
A
F_2'
X
$\vec{v}_0$
$\vec{v}_0$

b)

$$\alpha' \approx 50^0;\ \alpha'' \approx 70^0.$$

c)

$$\mathrm{v}_0 = \sqrt{KM\frac{2a_{\text{real}} - r_{0,\text{real}}}{a_{\text{real}} r_{0,\text{real}}}} = \sqrt{\frac{KM}{r_{0,\text{real}}}\cdot\frac{2a - r_0}{a}};$$

$$r_0 = 95\ \text{mm}\cdot S;\ S = \frac{30000\ \text{km}}{107\ \text{mm}};$$

$$r_{0,\text{real}} \approx 26636\ \text{km};\ 2a_{\text{real}} = 170\ \text{mm}\cdot S;$$

$$a = 85\ \text{mm}\cdot S;\ \ a_{\text{real}} = 23831{,}77\ \text{km};$$

$$2a = 170\ \text{mm};\ 2a_{\text{real}} = 47663{,}55\ \text{km};$$

$$\mathrm{v}_0 = \sqrt{\frac{6{,}67\cdot 10^{-11}\ \text{Nm}^2\text{kg}^{-2}\cdot 6\cdot 10^{24}\ \text{kg}}{26636\cdot 10^3\ \text{m}}\cdot\frac{170 - 95}{85}};$$

$$\mathrm{v}_0 = \sqrt{\frac{6{,}67\cdot 10^{-11}\cdot 6\cdot 10^{24}}{26636\cdot 10^3}\cdot\frac{170 - 95}{85}}\ \frac{\text{m}}{\text{s}} \approx 3640\ \frac{\text{m}}{\text{s}} \approx 3{,}6\ \frac{\text{km}}{\text{s}}.$$

d)

$$T^2 = \frac{4\pi^2}{K(M + m)}\cdot a_{\text{real}}^3;\ m << M;\ T^2 = \frac{4\pi^2}{KM}\cdot a_{\text{real}}^3;$$

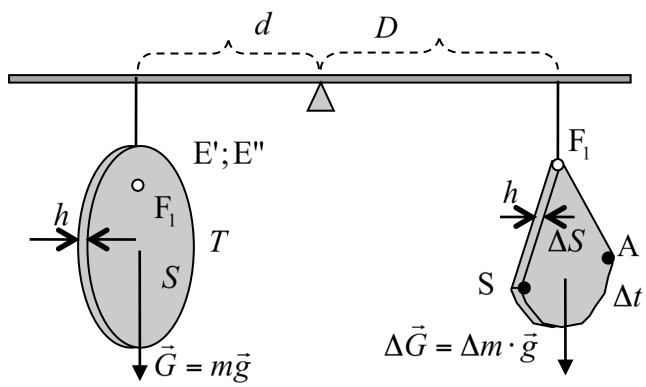
$$M = 6\cdot 10^{24}\ \text{kg};\ \ K = 6{,}67\cdot 10^{-11}\ \text{Nm}^2\text{kg}^{-2};$$

$$a_{\text{real}} = 23832\ \text{km};$$

$$T = 2\pi\sqrt{\frac{a_{\text{real}}^3}{KM}} = 2\cdot 3{,}14\sqrt{\frac{23832^3\cdot 10^9}{6{,}67\cdot 10^{-11}\cdot 6\cdot 10^{24}}}\ \text{s} \approx 36523\ \text{s};$$

$$T \approx 608\ \text{min} \approx 10\ \text{h}.$$

For each of two elipses the device has to be used



$$G \cdot d = \Delta G \cdot D;\ mg \cdot d = \Delta m \cdot g \cdot D;$$
$$m \cdot d = \Delta m \cdot D;\ \rho \cdot V \cdot d = \rho \cdot \Delta V \cdot D;$$
$$V \cdot d = \Delta V \cdot D;\ S \cdot h \cdot d = \Delta S \cdot h \cdot D;$$
$$S \cdot d = \Delta S \cdot D;\ \frac{\Delta S}{S} = \frac{d}{D}.$$

$$T \ldots\ldots\ldots\ldots\ldots\ldots\ldots S;$$
$$\Delta t \ldots\ldots\ldots\ldots\ldots\ldots \Delta S;$$
$$\Delta t = \frac{\Delta S}{S} \cdot T = \frac{d}{D} \cdot T;$$

For the projectile on elipse E', from measurements :

$$d = 12{,}8\text{ cm};\ D = 23\text{ cm};\ T = 10\text{ h};$$
$$\Delta t = \frac{d}{D} \cdot T = \frac{12{,}8\text{ cm}}{23\text{ cm}} \cdot 10\text{ h} \approx 5{,}56\text{ h};$$
$$d = 10{,}7\text{ cm};\ D = 25\text{ cm};$$
$$\Delta\tau = \frac{10{,}7\text{ cm}}{25\text{ cm}} \cdot 10\ h \approx 4{,}28\text{ h};$$
$$\Delta t = T - \Delta\tau = 5{,}72\text{ h};$$
$$\Delta t = \frac{5{,}56\text{ h} + 5{,}72\text{ h}}{2} = 5{,}64\text{ h}.$$

For the projectile on elipse E", from measurements:

$$d = 8{,}5\text{ cm};\ D = 22{,}3\text{ cm};\ T = 10\text{ h};$$
$$\Delta t = \frac{d}{D} \cdot T = \frac{8{,}5\text{ cm}}{22{,}3\text{ cm}} \cdot 10\text{ h} \approx 3{,}81\text{ h};$$
$$d = 13{,}5\text{ cm};\ D = 23\text{ cm};$$
$$\Delta\tau = \frac{13{,}5\text{ cm}}{23\text{ cm}} \cdot 10\ h \approx 5{,}86\text{ h};$$

$$\Delta t = T - \Delta\tau = 4{,}14\ \text{h};$$

$$\Delta t = \frac{3{,}81\ \text{h} + 4{,}14\ \text{h}}{2} = 3{,}975\ \text{h}.$$

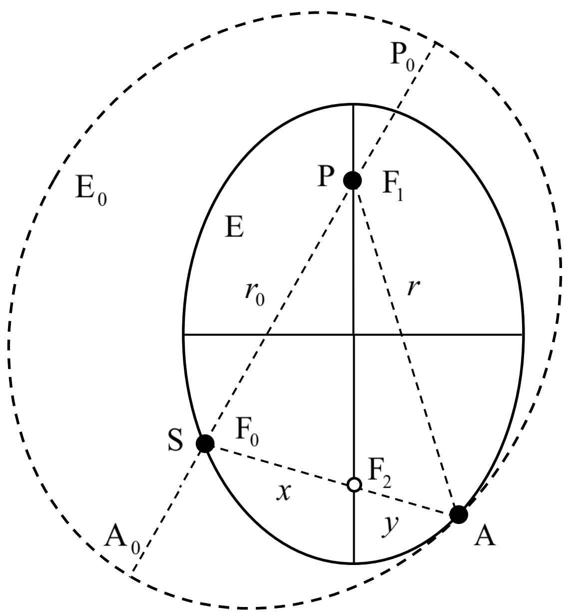
e) Modeling one wire on each sector SA:

$$l_{\text{SA}}^{'} = 183\ \text{mm};\ l_{\text{SA,real}}^{'} = l_{\text{SA}}^{'} S = 183\ \text{mm} \cdot \frac{30000\ \text{km}}{107\ \text{mm}} \approx 51308{,}41\ \text{km};$$

$$l_{\text{SA}}^{''} = 156\ \text{mm};\ l_{\text{SA,real}}^{''} = l_{\text{SA}}^{''} S = 156\ \text{mm} \cdot \frac{30000\ \text{km}}{107\ \text{mm}} \approx 43738{,}31\ \text{km}.$$

C.

a)



$$\text{PA} + \text{AS} = 2a_0;$$

$$r + x + y = 2a_0;$$

$$r_0 + x = 2a;\ r + y = 2a:$$

$$2a + 2a - r_0 = 2a_0;$$

$$2a_0 = 4a - r_0,$$

$$2a_0 = 222\text{ mm} - 74\text{ mm} = 148\text{ mm};$$

$$S = \frac{30000\text{ km}}{85\text{ mm}};$$

$$2a_{0,\text{real}} = 2a_0 S = 148\text{ mm}\cdot\frac{30000\text{ km}}{85\text{ mm}} \approx 52235{,}29\text{ km},$$

a-A:

$$2a_0 = 400\text{ mm} - 133\text{ mm} = 267\text{ mm};$$

$$S = \frac{30000\text{ km}}{153\text{ mm}};$$

$$2a_{0,\text{real}} = 2a_0 S = 267\text{ mm}\cdot\frac{30000\text{ km}}{153\text{ mm}} \approx 52352{,}94\text{ km};\ a_{0,\text{real}} = 26176{,}47\text{ km}.$$

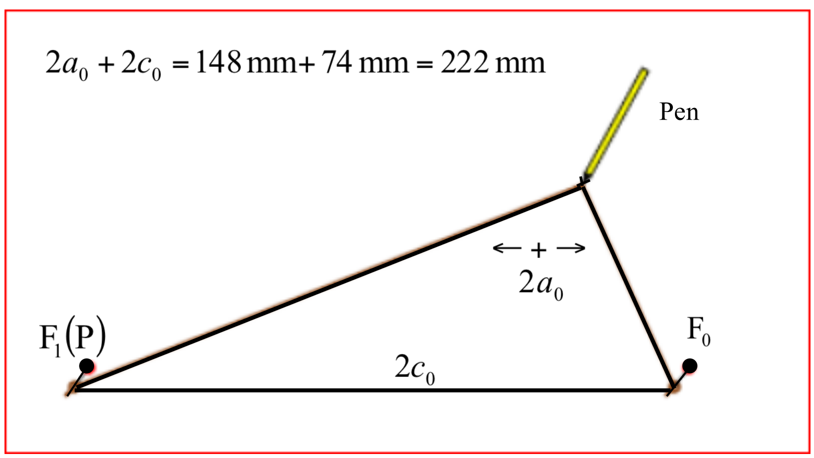
Security elipse .

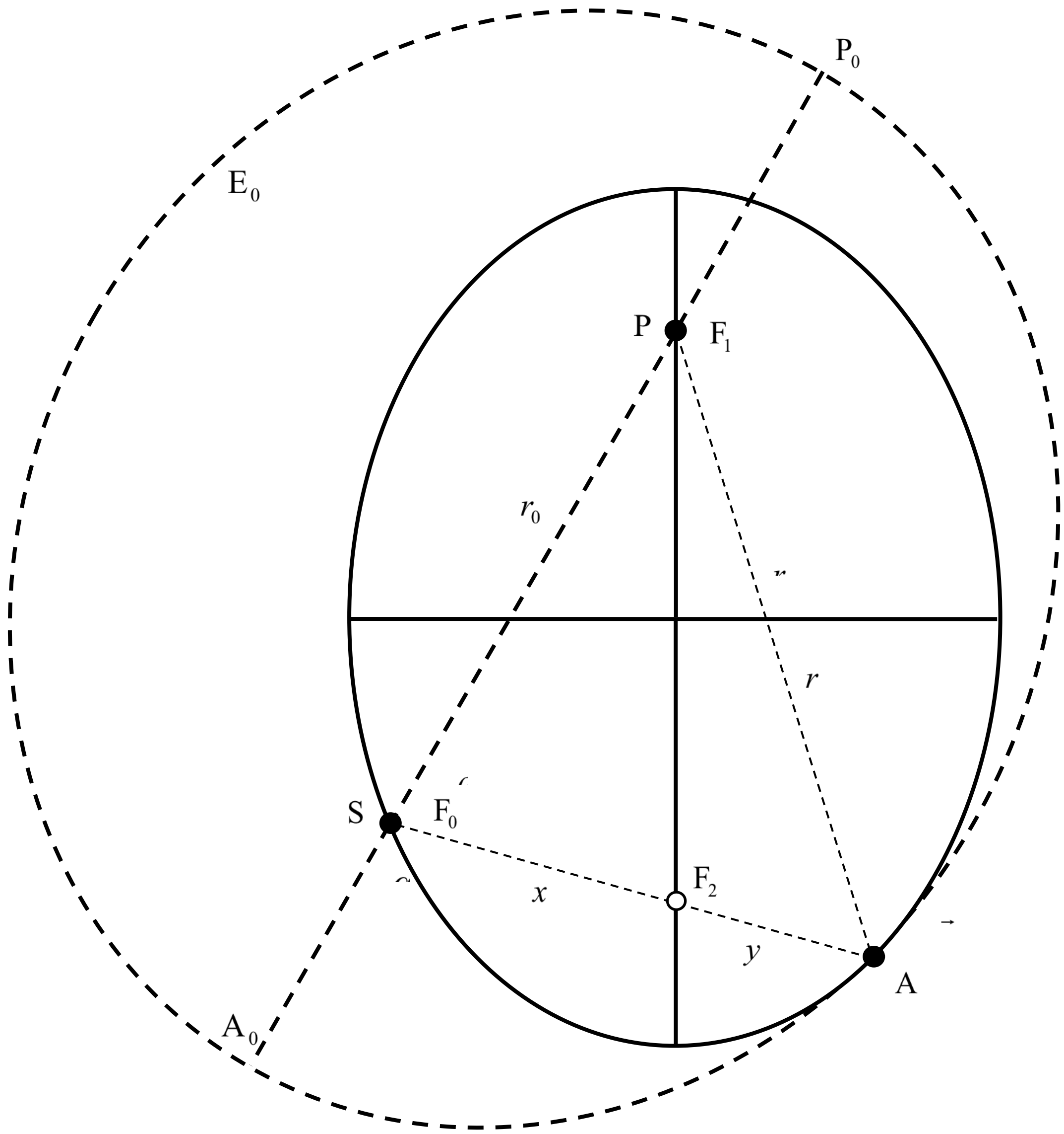
$$2c_{0,\text{real}} = r_{0,\text{real}} = 26078\text{ km};\ c_{0,\text{real}} = 13039\text{ km};$$

$$b_{0,\text{real}} = \sqrt{a_{0,\text{real}}^2 - c_{0,\text{real}}^2} \approx 22697{,}84\text{ km};$$

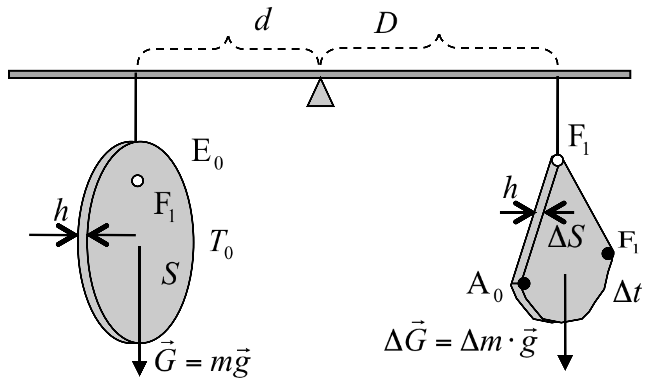
$$e_0 = \sqrt{1 - \frac{b_{0,\text{real}}^2}{a_{0,\text{real}}^2}} \approx 0{,}5.$$

Conturul elipsei de siguranță se trasează așa cum indică desenul din figura alăturată.





b)
c)



$$G \cdot d = \Delta G \cdot D;\ mg \cdot d = \Delta m \cdot g \cdot D;$$
$$m \cdot d = \Delta m \cdot D;\ \rho \cdot V \cdot d = \rho \cdot \Delta V \cdot D;$$
$$V \cdot d = \Delta V \cdot D;\ S \cdot h \cdot d = \Delta S \cdot h \cdot D;$$
$$S \cdot d = \Delta S \cdot D;\ \frac{\Delta S}{S} = \frac{d}{D}.$$

$$T_0 \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots S;$$
$$\Delta t \ldots\ldots\ldots\ldots\ldots\ldots \Delta S;$$
$$\Delta t = \frac{\Delta S}{S} \cdot T_0 = \frac{d}{D} \cdot T_0;$$
$$T_0^2 = \frac{4\pi^2}{KM} a_{0,\text{real}}^3;\ a_{0,\text{real}} = 26176{,}47 \text{ km};$$
$$M = 6 \cdot 10^{24} \text{ kg};\ \ K = 6{,}67 \cdot 10^{-11} \text{ Nm}^2\text{kg}^{-2};$$
$$T_0 = 2\pi \cdot a_{0,\text{real}} \cdot \sqrt{\frac{a_{0,\text{real}}}{KM}} \approx 42042{,}42 \text{ s};$$
$$T_0 \approx 700{,}7 \text{ min} \approx 11{,}67 \text{ h}.$$

$$d = 8{,}5 \text{ cm};\ D = 23 \text{ cm};\ \ T = 11{,}67 \text{ h};$$
$$\Delta\tau = \frac{d}{D} \cdot T = \frac{8{,}5 \text{ cm}}{23 \text{ cm}} \cdot 11{,}67 \text{ h} \approx 4{,}31 \text{ h};$$
$$\Delta t = T_0 - \Delta\tau = 7{,}36 \text{ h},$$